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General equation for op-amp sum/difference amps

Iain Sharp, 2 June 2013

I can never find on the web or in my text books the general equations for op-amps used as combined multi-input summing and difference amplifiers (ie they have several positive and negative inputs). It makes designing mixers for synthesizers annoyingly awkward as I have to rederive the equations each time. So, to save myself having to work everything out from scratch again, here are my derivations and notes on multi-input Op-Amp circuits. I will also take the opportunity to point out some interesting parts of the results.

Main results
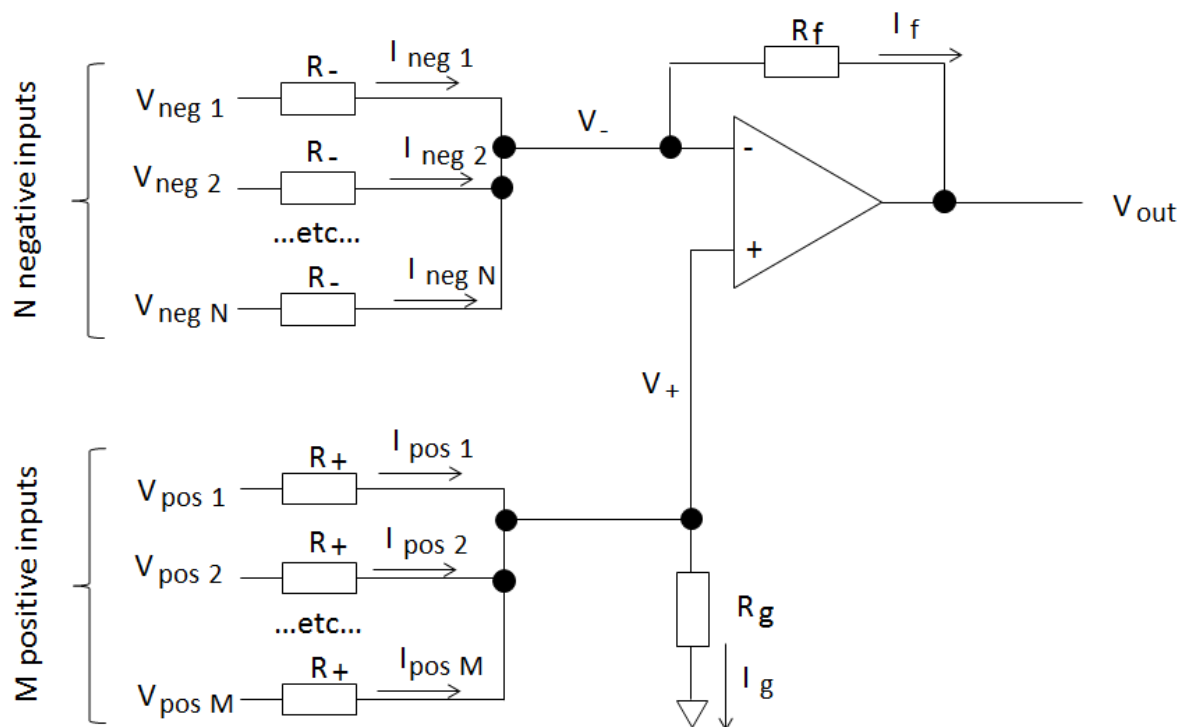
So, here's the setup:

We have an op-amp circuit with "N" negative inputs and "M" positive inputs as shown above. All the positive and negative inputs are identical.

For an ideal op-amp the output is:

$$V_{out} = \left(\frac{(NR_f + R_-)R_g}{(MR_g + R_+)R_-}\right) \sum_{m=1}^{M} V_{pos\,m} - \left(\frac{R_f}{R_-}\right) \sum_{n=1}^{N} V_{neg\,n}$$

Or, in other words the negative gain is:

$$G_- = \frac{R_f}{R_-}$$

The positive gain is:

$$G_+ = \frac{(NR_f + R_-)R_g}{(MR_g + R_+)R_-}$$

Positive and negative gain

The negative gain is nice and easy and only depends on the input and feedback resistors and not on any other variables, like the number of inputs. Why is this? Well the inverting input of the op-amp is a virtual ground and the voltage isn't changed by the negative inputs. Therefore the current through each negative input only depends on its input voltage. You can have as many or as few negative inputs as you like and it works the same.

The positive inputs are not in this lucky position! Voltages at the positive inputs change the voltage at both the inverting and non-inverting inputs of the op-amp. The non-inverting input voltage changes because of the voltage drop over Rg. The inverting input voltage changes due to the feedback action of the op-amp keeping the input voltages ideally identical. This means that currents flowing through all the input branches depend on the positive input voltages and hence the complicated positive gain equation.

Limits on positive gain values

Once the negative gain is set, this configuration limits the range of values of the positive gain depending on the number of positive and negative inputs. One particular example:

If the negative gain G- > 1 and number of negative inputs N < M, the number of positive inputs then G+ < G-.

To derive this then consider that the maximum positive gain is when the input resistors R+ = 0 (obvious from the circuit and also by inspection of the equation).

Special cases

There are a load of interesting special cases from this equation.

When N=M, and Rg=Rf and R+=R-, then:

$$V_{out} = \left(\frac{R_f}{R_-}\right) \sum_{m=1}^{M} V_{pos\,m} - \left(\frac{R_f}{R_-}\right) \sum_{n=1}^{N} V_{neg\,n}$$

When N=1 and M=0 (standard inverting op-amp), then:

$$V_{out} = -\left(\frac{R_f}{R_-}\right) V_{neg\,1}$$

When N=1, and M=1 and Vneg 1 =0 and R+=0 (standard non-inverting op-amp), then:

$$V_{out} = \left(1 + \frac{R_f}{R_-}\right) V_{pos\,1}$$

Derivation

For ideal op-amp with no input current, for the positive (non-inverting) input:

$$I_g = \sum_{m=1}^{M} I_{pos\,m}$$

$$\frac{V_+}{R_g} = \sum_{m=1}^{M} \frac{V_{pos\,m} - V_+}{R_+}$$

$$\frac{R_+ V_+}{R_g} = \sum_{m=1}^{M} V_{pos\,m} - \sum_{m=1}^{M} V_+$$

$$\sum_{m=1}^{M} V_{pos\,m} = MV_+ + \frac{R_+ V_+}{R_g}$$

$$V_+ \left(\frac{MR_g + R_+}{R_g}\right) = \sum_{m=1}^{M} V_{pos\,m}$$

$$V_+ = \frac{R_g \sum_{m=1}^{M} V_{pos\,m}}{MR_g + R_+}$$

For ideal op-amp with no input current, for the negative (inverting) input:

$$I_f = \sum_{n=1}^{N} I_{neg\,n}$$

$$\frac{V_- - V_{out}}{R_f} = \sum_{n=1}^{N} \frac{V_{neg\,n} - V_-}{R_-}$$

$$R_- \left(\frac{V_- - V_{out}}{R_f}\right) = \sum_{n=1}^{N} V_{neg\,n} - NV_-$$

$$NV_- + R_- \left(\frac{V_- - V_{out}}{R_f}\right) = \sum_{n=1}^{N} V_{neg\,n}$$

$$\frac{R_f N V_- + R_- V_- + R_- V_{out}}{R_f} = \sum_{n=1}^{N} V_{neg\,n}$$

$$\frac{V_-(R_f N + R_-)}{R_f} = \sum_{n=1}^{N} V_{neg\,n} + \frac{V_{out} R_-}{R_f}$$

$$V_- = \frac{R_f \sum_{n=1}^{N} V_{neg\,n} + V_{out} R_-}{(N R_f + R_-)}$$

Now, for an ideal op-amp in normal operating mode:

$$V_- = V_+$$

Substituting from above:

$$\frac{R_f \sum_{n=1}^{N} V_{neg\,n} + V_{out} R_-}{(N R_f + R_-)} = \frac{R_g \sum_{m=1}^{M} V_{pos\,m}}{M R_g + R_+}$$

$$R_f \sum_{n=1}^{N} V_{neg\,n} + V_{out} R_- = \frac{(N R_f + R_-) R_g}{(M R_g + R_+)} \sum_{m=1}^{M} V_{pos\,m}$$

$$V_{out} = \left(\frac{(N R_f + R_-) R_g}{(M R_g + R_+) R_-}\right) \sum_{m=1}^{M} V_{pos\,m} - \left(\frac{R_f}{R_-}\right) \sum_{n=1}^{N} V_{neg\,n}$$